Edge-colouring permutation graphs

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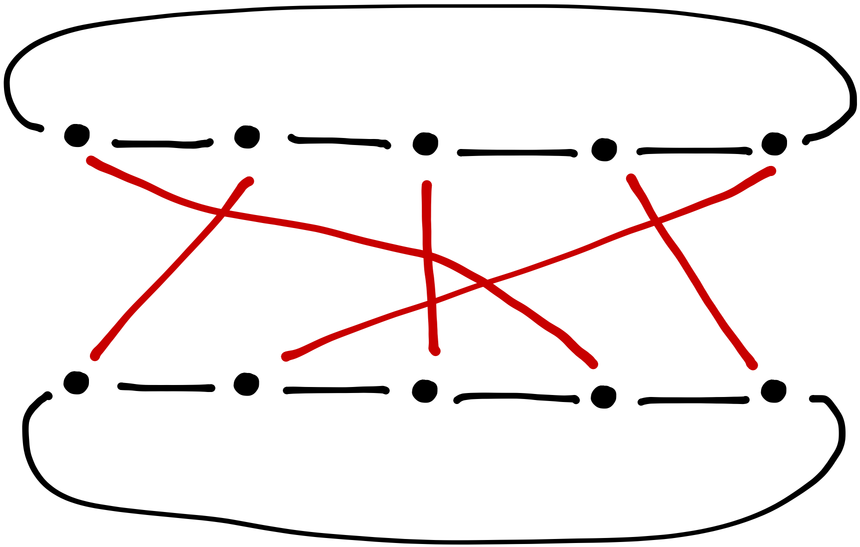
Permutation graphs

= cubic graphs with a 2-factor of two chordless cycles



Permutation snarks

- chromatic index 3 or 4
- infinitely many with $\chi' = 4$ ($\rightarrow$ permutation snarks)
- clearly on $2n$ vertices, n odd

Refuted conjecture (Zhang):
P_{10} is the only cyclically 5-edge-connected permutation snark.

- other such snarks constructed by Hägglund and Hoffmann-Ostenhof

Problems we'll consider

- the 4-flow conjecture
- orders of permutation snarks
- Berge-Fulkerson and related conjectures

The 4-flow conjecture

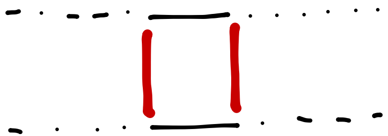
Conjecture (Tutte):

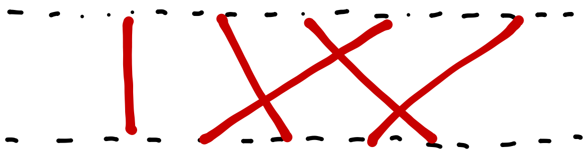
Every bridgeless graph with no nowhere-zero 4-flow contains a Petersen minor.

EASY for permutation graphs.

Theorem (Ellingham 84):

A permutation graph contains either an M-C_4, or an M-P_{10}.

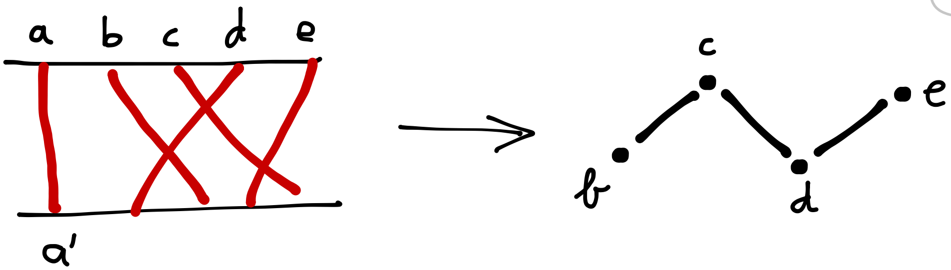




Theorem (TK, Sereni, Yilma 13):
If G is a permutation graph on ≥ 6 vertices and $e \in E(M)$ is contained in every M-C_4, then e is contained in an M-P_{10}.

Corollary: A permutation graph with ≥ 40 vertices and no M-C_4 contains $\geq \frac{n}{2} - 4$ copies of M-P_{10}.
(this is tight up to a constant factor)

- interesting link to cographs:



no M-P_{10} containing $aa' \Rightarrow H_a$ is a cograph

auxiliary graph H_a

Orders of permutation snarks

Brinkmann, Goedgebeur, Hägglund, Markström 13:

order	# permutation snarks
10	1
14	0
18	2
22	0
26	64
30	0
34	10 771

Problem: Do all permutation snarks have 2 (mod 8) vertices?

Berge-Fulkerson Conjecture

Conjecture (Fulkerson):
Every bridgeless cubic graph admits 6 perfect matchings such that each edge is contained in two of them.

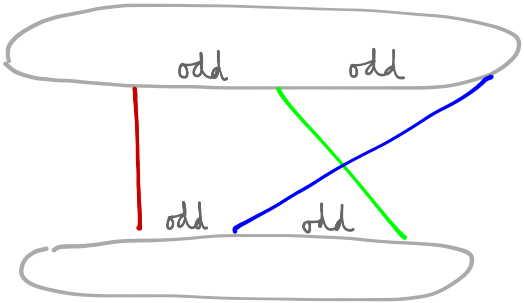
equivalent but not when restricted to permutation graphs!

NOTHING KNOWN for permutation graphs.

Conjecture (Berge):
The edges of any bridgeless cubic graph can be covered by 5 PMs.

TRUE for permutation graphs:

Theorem (Fouquet, Vanherpe 09):
4 PMs suffice except in P_{10}.



Related problems

Conjecture (Fan, Raspaud):

Every bridgeless cubic graph contains 3 PMs with empty intersection.

EASY from the Fouquet-Vanherpe result.

Conjecture (Patel; TK, Král', Norin):

Every bridgeless cubic graph contains 3 PMs covering $\frac{4}{5}$ of the edges.

EASY for the same reason (even with $5/6$).

Conjecture (Jaeger; Petersen colouring)

The edges of any bridgeless cubic graph can be coloured with edges of P_{10} such that adjacent edges have adjacent colours.

NOTHING KNOWN.